# Hyperfine structure of the ground state in the helium-muonic atoms

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# Outline

# Motivation



Theoretical investigations of the energy spectrum have achieved significant successes in two approaches:

1)The first approach was based on perturbation theory for the Schrodinger equation. In this case, there exists an analytic form for a three-body wave function in the initial approach. On this basis various corrections of hyperfine splitting were made.

2)The other approach was based on the variational method in quantum mechanics. It allowed to numerically compute bound energy levels of a three-body system with very high accuracy. To find low-lying energy levels with high accuracy one needs to consider various corrections of an interaction operator of particles. First of all, these corrections are related to the effect of recoil, nuclear structure and vacuum polarization.

# Hamiltonian of the muonic helium systems (in a.u.)



$$H = -\frac{1}{2}\left[\nabla_e^2 + \frac{1}{m_\mu}\nabla_\mu^2 + \frac{1}{m_N}\nabla_N^2\right] + \frac{1}{r_{e\mu}} - \frac{2}{r_{eN}} - \frac{2}{r_{\mu N}} \qquad (1)$$

$$r_{\mu N} = |\vec{r}_\mu - \vec{r}_N|, \qquad m_\mu = 206.768262,$$
$$r_{eN} = |\vec{r}_e - \vec{r}_N|, \qquad M_{^3He^{2+}} = 5495.8852,$$
$$r_{e\mu} = |\vec{r}_e - \vec{r}_\mu|, \qquad M_{^4He^{2+}} = 7294.2996.$$

# Variational Method



The wave functions are taken in the form:

$$\psi(r_{\mu N}, r_{eN}, r_{e\mu}) = \sum_n C_n e^{-\alpha_n r_{\mu N} - \beta_n r_{eN} - \gamma_n r_{e\mu}}, \tag{2}$$

The nonlinear parameters from (2) are generated using the following simple formulas:

$$\alpha_n = \left[\left[\frac{1}{2}n(n+1)\sqrt{p_\alpha}\right](A_2 - A_1) + A_1\right],$$

$$\beta_n = \left[\left[\frac{1}{2}n(n+1)\sqrt{p_\beta}\right](B_2 - B_1) + B_1\right],$$

$$\gamma_n = \left[\left[\frac{1}{2}n(n+1)\sqrt{p_\gamma}\right](C_2 - C_1) + C_1\right].$$

where $[x]$ designates the fractional part of $x$, $p_\alpha$ are some prime numbers, and $[A_1, A_2]$ are real variational intervals, which need to be optimized. Parameters $\alpha_n$, $\beta_n$ and $\gamma_n$ are obtained in a similar way.

# Results



The convergence of the total energies in atomic units for the ground $1s\mu 1se$-states in the helium-muonic atoms. $N$ is the total number of basis functions used in calculations.

| $(N)$ | $^3\mathrm{He}^{2+}\mu^- e^-$ | $^4\mathrm{He}^{2+}\mu^- e^-$ |
|---|---|---|
| 2000 | -399.042 336 832 862 534 827 027 433 | -402.637 263 035 135 454 018 960 573 |
| 2500 | -399.042 336 832 862 534 827 039 305 | -402.637 263 035 135 454 018 972 984 |
| 3000 | -399.042 336 832 862 534 827 041 147 | -402.637 263 035 135 454 018 973 292 |
| 3500 | -399.042 336 832 862 534 827 041 500 | -402.637 263 035 135 454 018 974 187 |
| 4000 | -399.042 336 832 862 534 827 041 545 | -402.637 263 035 135 454 018 974 468 |
| 4500 | -399.042 336 832 862 534 827 041 560 | -402.637 263 035 135 454 018 974 488 |

The convergence of the expectation values for the delta functions for various pairs of particles. $N$ is the total number of basis functions used in calculations.

| $N$ | $^3\mathrm{He}^{2+}\mu^- e^-$ | | | $^4\mathrm{He}^{2+}\mu^- e^-$ |
|---|---|---|---|---|
| | $\langle\delta(\mathbf{r}_{N\mu})\rangle$ | $\langle\delta(\mathbf{r}_{Ne})\rangle$ | $\langle\delta(\mathbf{r}_{\mu e})\rangle$ | $\langle\delta(\mathbf{r}_{\mu e})\rangle$ |
| 2000 | 20 149 938.845 | 0.320 611 550 99 | 0.313 682 320 01 | 0.313 760 536 34 |
| 2500 | 20 149 938.845 | 0.320 611 551 24 | 0.313 682 320 01 | 0.313 760 536 39 |
| 3000 | 20 149 938.845 | 0.320 611 551 42 | 0.313 682 320 00 | 0.313 760 536 38 |
| 3500 | 20 149 938.845 | 0.320 611 551 51 | 0.313 682 320 00 | 0.313 760 536 38 |
| 4000 | 20 149 938.845 | 0.320 611 551 56 | 0.313 682 319 99 | 0.313 760 536 37 |
| 4500 | 20 149 938.845 | 0.320 611 551 57 | 0.313 682 319 99 | 0.313 760 536 37 |

# Hyperfine structure of muonic helium atoms



For $S$ states the spin dependent term of the Breit-Pauli Hamiltonian is

$$\boxed{H_{\rm HFS} = -\frac{8\pi}{3}\boldsymbol{\mu}_N\boldsymbol{\mu}_\mu\,\delta(\mathbf{r}_{N\mu}) - \frac{8\pi}{3}\boldsymbol{\mu}_e\boldsymbol{\mu}_\mu\,\delta(\mathbf{r}_{e\mu}) - \frac{8\pi}{3}\boldsymbol{\mu}_N\boldsymbol{\mu}_e\,\delta(\mathbf{r}_{Ne}).} \qquad (3)$$

For $^4$He since the spin of nucleus is zero the Hamiltonian is simplified:

$$H_{\rm HFS} = -\frac{8\pi}{3}\boldsymbol{\mu}_e\boldsymbol{\mu}_\mu\,\langle\delta(\mathbf{r}_{e\mu})\rangle = E_1(s_e, s_\mu),$$

where

$$E_1 = -4464.55(60)\ \text{MHz}.$$

For $^3$He the effective HFS Hamiltonian has three terms:

$$H_{\mathrm{HFS}} = E_1(s_e, s_\mu) + E_2(s_h, s_\mu) + E_3(s_h, s_e),$$

where

$$E_1 = -4463.44(24)\ \text{MHz},$$
$$E_2 = -331846.(16.)\ \text{GHz},$$
$$E_3 = -1091.750(58)\ \text{MHz}.$$

The coupling scheme is $\mathbf{F} = \mathbf{s}_h + \mathbf{s}_\mu$, $\mathbf{J} = \mathbf{F} + \mathbf{s}_e$, and the spin state is denoted as $|FJ\rangle$.
Diagonalization of the effective HFS Hamiltonian gives the splitting:

$$\Delta\nu(\chi_{|0,1/2\rangle}) = 248884463.3 \text{ MHz}\,,$$

$$\Delta\nu(\chi_{|1,1/2\rangle}) = -82962876.6 \text{ MHz}\,,$$

$$\Delta\nu(\chi_{|1,3/2\rangle}) = -82958710.2 \text{ MHz}\,.$$

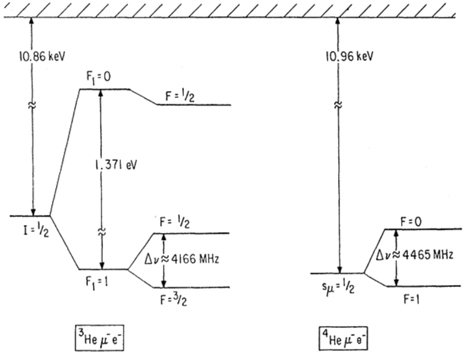
and for the difference of the lower state ($|F = 1\rangle$):

$$\delta\nu(\chi_{|1,3/2-1/2\rangle}) = 4166.39 \text{ MHz}\,.$$

# Spectra of the muonic helium atoms



Keh-Ning Huang and Vernon W. Hughes "Theoretical hyperfine structure of the muonic $^3$He and $^4$He atoms". Physical review A, 1982.



# Summary



1) We have calculated the ground state energies of the muonic helium with a record precision by using the varational method.

2) The differences between corresponding levels of hyperfine structure are determined to very high numerical accuracy. In particular, we have found that the hyperfine structure splitting in the ground state of the $^4He^{2+}\mu^- e^-$ atom is $\Delta\nu \approx 4464.55(60)$ MHz , while analogous splitting for the $^3He^{2+}\mu^- e^-$ atom is $\Delta\nu \approx 4166.39$ MHz

# Thank You!