Michel parameters in radiative μ and τ decays. Higher-order radiative corrections to low-energy *ep* scattering

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Outline

- Motivation
- Normalization of Michel parameters
- Comments on muon decay asymmetry
- Model-independent description of radiative muon decay
- Higher order corrections to elastic *ep* scattering

Motivation

Low-energy high-precision experiments provide important tests of the SM

Muon decay is one of the key-stones of particle physics

A lot of new data on τ decays is coming from LHC and modern e^+e^- colliders

The puzzle of the proton charge radius motivates for new studies both thoretical and experimental

Effects of higher orders in α_{QED} and/or in small mass ratio should be taken into account

The report is based on
[1] *Michel parameters in radiative muon decay* JHEP 1609 (2016) 109;
[2] *On higher order radiative corrections to elastic electron-proton scattering*, Eur.Phys.J.C 75 (2015) 603

Normalization of Michel parameters (I)

The Fermi coupling constant is know with 0.5ppm precision:

$$G_{\rm Fermi} = 1.166\ 378\ 7(6)\ \cdot\ 10^5\ {\rm GeV}^2$$

It is defined from the total muon decay width:

$$\Gamma_\mu = \frac{G^2_{\rm Fermi} m_\mu^5}{192\pi^3} f(r) \left\{ 1 + H_1(r)\frac{\hat{\alpha}(m_\mu)}{\pi} + H_2(r)\left(\frac{\hat{\alpha}(m_\mu)}{\pi}\right)^2 \right\}$$

$$\hat{\alpha}(m_\mu) \approx \frac{1}{139.901}, \qquad r \equiv \frac{m_e}{m_\mu}$$

$$f(r) = 1 - 8r^2 + 8r^6 - r^8 - 12r^2 \ln r^2$$

N.B. Function $f(r)$ is factorized by hand

$H_2(r)$ — [A.Pak, A.Czarnecki, PRL '2008]

Normalization of Michel parameters (II)

Within effective model-independent approach

$$\mathcal{M} = 4\frac{G_0}{\sqrt{2}} \sum_{\substack{\gamma=\mathrm{S,V,T} \\ \epsilon,\omega=\mathrm{R,L}}} g^{\gamma}_{\epsilon\omega} \langle \bar{l}_\epsilon | \Gamma^\gamma | \nu_l \rangle \langle \bar{\nu}_\tau | \Gamma_\gamma | \tau_\omega \rangle \Longrightarrow$$

$$\Gamma_\mu = \frac{G_0^2 m_\mu^5}{192\pi^3} \left\{ f(r) \cdot N + 4\, r\, g(r) \cdot E \right\} + \mathcal{O}(\alpha)$$

$$g(r) = 1 + 9r^2 - 9r^4 - r^6 + r^2(1 + r^2) \ln r^2$$

N and E are bilinear combinations of couplings $g^{\gamma}_{\epsilon\omega}$

$E/N \equiv \eta$ is one of the Michel parameters, $\eta = 0.057(34)$ [PDG]

The standard normalization $G_{\mathrm{Fermi}} \equiv G_0 \cdot N$

N.B. Normalization is perfect if $r \to 0$ or $\eta \to 0$

Normalization of Michel parameters (III)

$$
\begin{aligned}
N &= \frac{1}{4}\left(|g^S_{LL}|^2 + |g^S_{LR}|^2 + |g^S_{RL}|^2 + |g^S_{RR}|^2\right) \\
&+ \left(|g^V_{LL}|^2 + |g^V_{LR}|^2 + |g^V_{RL}|^2 + |g^V_{RR}|^2\right) + 3\left(|g^T_{LR}|^2 + |g^T_{RL}|^2\right) \\
E &= \frac{1}{2}\mathrm{Re}\left(g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR}) + g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL})\right) \\
&+ \frac{1}{2}\mathrm{Re}\left(g^V_{RR}\, g^{S*}_{LL} + g^V_{LL}\, g^{S*}_{RR}\right)
\end{aligned}
$$

Standard normalization: $N \equiv 1$

Modified normalization: $N^* = N + 4r\frac{g(r)}{f(r)}E \equiv 1$

is used (not explicitly) for τ decays

The corresponding change of Michel parameters

$$
\rho \to \rho^* = \frac{\rho}{1 + 4r\eta\frac{g(r)}{f(r)}}, \qquad \eta \to \eta^* = \frac{\eta}{1 + 4r\eta\frac{g(r)}{f(r)}}, \qquad \ldots
$$

Muon decay spin asymmetry (1)

$$A \equiv \frac{1}{\Gamma_{\mathrm{tot}}} \left[\int_0^{+1} dc \frac{d\Gamma}{dc} - \int_{-1}^{0} dc \frac{d\Gamma}{dc} \right], \qquad \Gamma_{\mathrm{tot}} \equiv \int_{-1}^{+1} dc \frac{d\Gamma}{dc}$$

where c is the cosine of the angle between the electron momentum and muon spin

N.B.1. The spin asymmetry can be measured rather accurately
$\Longrightarrow$ experimental issues $\quad + \quad$ higher order corrections

N.B.2. The spin asymmetry would provide additional information on weak interactions in comparison to the total width.

Muon decay spin asymmetry (2)

The spin asymmetry is sensitive to Michel paramters ξ and $\xi\delta$:

$$A = P_\mu \xi \cdot f_\xi(r) + P_\mu \xi\delta \cdot f_\delta(r),$$

$$f_\xi(r) = -\frac{1}{3} + \frac{20}{3}r^2 - \frac{64}{3}r^3 + 30r^4 - \frac{64}{3}r^5 + \frac{20}{3}r^6 - \frac{1}{3}r^8$$

$$f_\delta(r) = -\frac{80}{9}r^2 + \frac{128}{3}r^3 - 80r^4 + \frac{640}{9}r^5 - \frac{80}{3}r^6 + \frac{16}{6}r^8$$

where P_μ is the muon polarization degree

One-loop corrections within the Fermi model, ($\xi = 1$ and $\xi\delta = 3/4$) were computed in [A.A. PLB'2002]:

$$A^{(1)} = P_\mu \frac{\alpha}{2\pi}\left[-\frac{617}{108} + \frac{14}{3}\zeta(2) - \frac{8}{3}r + r^2\left(-32 + 16\zeta(2) + \frac{2}{3}\ln r^2\right) + \mathcal{O}(r^3)\right]$$

Muon decay spin asymmetry (3)

Second order QED corrections to the spin asymmetry were computed in [F. Caola, A. Czarnecki, Y. Liang, K. Melnikov, R. Szafron, PRD '2014]

$$A^{(2)} = A_0 \frac{\bar{\alpha}(m_\mu)}{\pi} \cdot 11.2(1), \qquad A_0 = -\frac{1}{3} P_\mu$$

N.B.1. $A^{(2)}$ is of the same order as the term $\sim r$ in $A^{(1)}$.

N.B.2. Treatment of contribution of $\mu \to e^- e^+ e^- \nu_\mu \bar{\nu}_e$ (real pair production) with "QCD-like re-combination to a jet" doesn't correspond to conditions of the TWIST experiment

Radiative muon decay (1)

Studies of radiaitve decays $\mu \to e\nu_\mu\bar{\nu}_e\gamma$, $\tau \to \mu\nu_\tau\bar{\nu}_\mu\gamma$, and $\tau \to e\nu_\tau\bar{\nu}_e\gamma$ provide additional information on Michel parameters and tests of lepton universality

Analysis of τ decays are quite accurate, see e.g. [Belle Coll. arXiv:1609.08280], more data and results are coming

Exact dependence on the charged lepton masses for the radiative decays in model-independent approach was presented recently in [A.A., T.Kopylova, JHEP'2016]

N.B.1. Effects supressed by mass ratios are crucial for $\tau \to \mu \ldots$ decays, since $m_\mu/m_\tau \approx 1/17$ is not so small

N.B.2. A misprint in [C. Fronsdal and H. Uberall, PRD'59] is corrected.

Radiative muon decay (2)

The differential width

$$\frac{d\Gamma(\mu^\pm \to e^\pm \bar{\nu}\nu\gamma)}{dx\, dy\, d\Omega_e\, d\Omega_\gamma} = \Gamma_0 \frac{\alpha_{\rm QED}}{64\pi^3} \frac{\beta_e}{y} \Big[F(x,y,d) \mp \beta_e P_\mu \cos\theta_e G(x,y,d)$$

$$\mp P_\mu \cos\theta_\gamma H(x,y,d) \Big], \qquad \Gamma_0 = \frac{G^2_{\rm Fermi} m^5_\mu}{192\pi^3}, \qquad \beta_e = \sqrt{1 - \frac{m_e^2}{E_e^2}}$$

$$d = 1 - \beta_e \cos\theta_{e\gamma}, \qquad F(x,y,d) = \sum_{k=1}^{5} \left(\frac{m_e}{m_\mu} \right)^k F^{(k)}(x,y,d), \ldots$$

Sensitivity to different Michel parameters ρ, η, $\bar{\eta}$, ξ, δ, κ, α, β depends on event selection.

Normalization choice is crucial for extraction of η value.

Radiative muon decay (3)

Let's look at terms linear in the mass ratio:

$$F^{(1)} = \beta\biggl(\frac{24x(x+y-1)}{d} - \frac{4y^2}{d} + 4xy^2 - 12x(xy+x+y-1) \\ -x^2y^2d + 6x^2yd \biggr)$$

$$G^{(1)} = 0, \qquad H^{(1)} = 0, \qquad \beta = -4\mathrm{Re}(g^V_{RR} g^{S*}_{LL} + g^V_{LL} g^{S*}_{RR})$$

Radiative muon decay (4)

Integrated radiative decasy width for $\tau \to \mu\nu_\tau\bar{\nu}_\mu$

A) $E_\gamma > 10$ MeV, standard normalization

$$\Gamma = \Gamma_0 \Big[0.0172 \cdot N + 0.0044 \cdot \rho - 0.0013 \cdot \eta + 0.0003 \cdot \alpha - 0.0006 \cdot \beta \Big]$$

B) $E_\gamma > 10$ MeV, alternative normalization

$$\Gamma = \Gamma_0^* \Big[0.0172 \cdot N^* + 0.0044 \cdot \rho^* - 0.0051 \cdot \eta^* + 0.0003 \cdot \alpha^* - 0.0006 \cdot \beta^* \Big]$$

Summary for Part I

1. Alternative normalization of Michel parameters is discussed

2. Muon decay spin asymmetry and mass-ratio corrections to it are discussed

3. Complete mass dependence of radiative muon (tau) decay in model-independent approach is presented

4. New experiments on muon and leptonic tau decays have a high potential for new physics searches and lepton universality tests

Part II

Motivation for Part II

- The proton charge radius puzzle is one of a few cases where SM predictions $\neq$ observations (5.6σ)
- Actually, we have a difference between the results of two independent analyses of exp. data, but both do include theoretical input
- Obviously to resolve to problem, one has to check everything:
  — look for exp. bugs or underestimated uncertainties;
  — look for problems in data analysis;
  — verify assumptions on charge distribution (pion clouds . . .);
  — re-consider theoretical predictions with higher order effects;
  — look for possible new physics contributions
- Experiments on atomic spectra look more save, but . . .
- Here: effects of radiative corrections in elastic *ep* scattering
- Concrete (simplified) event selection of MAMI is applied

The MAMI experiment (I)

Mainz Microtron experimental set-up:

— the electron beam energy $E_e \equiv E \lesssim 855$ MeV (1.6 GeV)

— momentum transfer range: $0.003 < Q^2 < 1$ GeV^2

— the outgoing electron energy $E_e{}' \equiv E' > E_e - \Delta E$

— no any other condition: neither on emitted particles nor on angles

— experimental precision (point-to-point) $\simeq 0.37\% \quad \to \quad 0.1\%$ (?)

$\Rightarrow$ all effects at least of the 10^{-4} order should be taken into account. That is not a simple task in any case

N.B. $E_e^2 \gg m_e^2, \quad Q^2 \gg m_e^2, \quad (\Delta E)^2 \gg m_e^2$

Ref.: J.C. Bernauer et al. [A1 Coll.] PRC 90 (2014) 015206

The MAMI experiment (II)

The Born cross section is written via the Sachs form factors:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \left[\frac{G_E^2\left(Q^2\right)+\tau G_M^2\left(Q^2\right)}{1+\tau} + 2\tau G_M^2\left(Q^2\right)\tan^2\frac{\theta}{2}\right]$$
$$= \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \frac{\varepsilon G_E^2 + \tau G_M^2}{\varepsilon\left(1+\tau\right)}, \qquad \tau = \frac{Q^2}{4m_P^2}, \quad \varepsilon = \ldots$$

The proton charge radius is defined then via

$$\left\langle r^2\right\rangle = -\frac{6}{G_E\left(0\right)} \left.\frac{\mathrm{d}G_E\left(Q^2\right)}{\mathrm{d}Q^2}\right|_{Q^2=0}$$

i.e., from the slope of the E form factor at $Q^2=0$

Types of RC to elastic *ep* scattering

— Virtual (loop) and/or real emission

— QED, QCD, and (electro)weak effects

— Perturbative and/or non-perturbative contributions

— Perturbative QED effects in $\mathcal{O}(\alpha)$, $\mathcal{O}\left(\alpha^2\right)$, ...

— Leading and next-to-leading logarithmic approximations

— Corrections to the electron line, to the proton line, and their interference

— Vacuum polarization, vertex corrections, double photon exchange etc.

First order QED RC (I)

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_1 = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 (1+\delta)$$

The $\mathcal{O}(\alpha)$ QED RC with point-like proton are well known:
Refs.: see e.g. L.C. Maximon & J.A. Tjon, PRC 2000

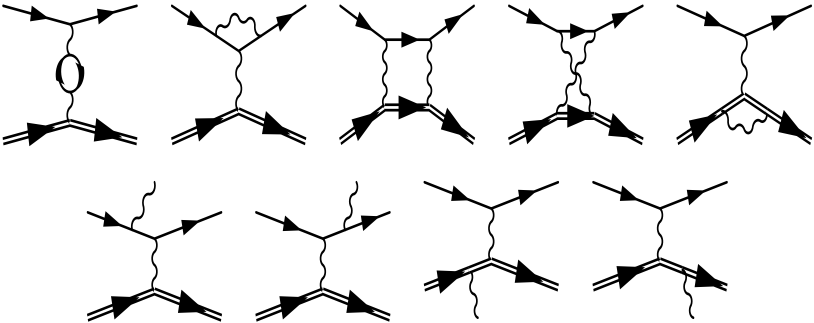
Virtual RC: Vacuum polarization, vertex, and box Feynman diagrams

Real RC: emission off the initial and final electrons and protons

N.B.1. UV divergences are regularized and renormalized;

N.B.2. IR divergences cancel out in sum of virtual and real RC

First order QED RC (II)



Ref.: J.C. Bernauer et al. [A1 Coll.] PRC 90 (2014) 015206

Size of RC

The problem has several small and large parameters to be used in expansions:

- $\alpha/(2\pi) \approx 0.001$
- $(\alpha/(2\pi))^2 \approx 10^{-6}$
- $L \equiv \ln(Q^2/m_e^2) \approx 16$ is the large log for $Q^2 = 1$ GeV^2
- $\ln(\Delta) \sim 5$, where $\Delta = \Delta E_e/E_e \ll 1$

N.B. Some $\mathcal{O}\left(\alpha^2\right)$ corrections are enhanced with 2nd, 3rd or even 4th power of large logs. So, they should be treated with care.

Vacuum polarization in one-loop

$$\delta^{(1)}_{\rm vac} = \frac{\alpha}{\pi}\frac{2}{3}\left\{\left(v^2 - \frac{8}{3}\right) + v\frac{3-v^2}{2}\ln\left(\frac{v+1}{v-1}\right)\right\}$$

$$\overset{Q^2 \gg m_l^2}{\longrightarrow} \frac{\alpha}{\pi}\frac{2}{3}\left\{-\frac{5}{3} + \ln\left(\frac{Q^2}{m_l^2}\right)\right\}, \quad v = \sqrt{1 + \frac{4m_l^2}{Q^2}}, \quad l = e, \mu, \tau$$

Two ways of re-summation:

1) geometric progression

$$\Rightarrow \alpha(Q^2) = \frac{\alpha(0)}{1 - \Pi(Q^2)}, \quad \Pi(Q^2) = \frac{1}{2}\delta^{(1)}_{\rm vac} + \ldots$$

2) exponentiation

$$\alpha(Q^2) = \alpha(0)e^{\delta^{(1)}_{\rm vac}/2}$$

the latter option was used by A1 Coll.

Other $\mathcal{O}(\alpha)$ effects

$$\delta^{(1)}_{\rm vertex} = \frac{\alpha}{\pi}\left\{\frac{3}{2}\ln\left(\frac{Q^2}{m^2}\right) - 2 - \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right) + \frac{\pi^2}{6}\right\}$$

$$\begin{aligned}\delta^{(1)}_{\rm real} &= \frac{\alpha}{\pi}\left\{\ln\left(\frac{(\Delta E_s)^2}{E\cdot E'}\right)\left[\ln\left(\frac{Q^2}{m^2}\right) - 1\right] - \frac{1}{2}\ln^2\eta + \frac{1}{2}\ln^2\left(\frac{Q^2}{m^2}\right)\right. \\ &- \left.\frac{\pi^2}{3} + {\rm Sp}\left(\cos^2\frac{\theta_e}{2}\right)\right\}, \quad \eta = \frac{E}{E'}, \quad \Delta E_s = \eta\cdot\Delta E'\end{aligned}$$

Interference δ_1 and radiation off proton δ_2 do not contain large logs.
A1 Coll. applied RC in the exponentiated form:

$$\left(\frac{{\rm d}\sigma}{{\rm d}\Omega}\right)_{\rm exp}(\Delta E') = \left(\frac{{\rm d}\sigma}{{\rm d}\Omega}\right)_0 e^{\delta_{\rm vac}+\delta_{\rm vertex}+[\delta_{\rm real}+\delta_1+\delta_2](\Delta E')}$$

Higher order effects are partially taken into account by exponentiation.
Remind the Yennie-Frautschi-Suura theorem

Multiple soft photon radiation

Exponentiation corresponds to independent emission of soft photons, while the cut on the total lost energy leads to sizable shifts.

For two photons:

$$\mathrm{e}^{\delta_{\mathrm{soft}}} \to \mathrm{e}^{\delta_{\mathrm{soft}}} - \left(\frac{\alpha}{\pi}\right)^2 \frac{\pi^2}{3} (L-1)^2$$

at $Q^2 = 1$ GeV^2 this gives $-3.5 \cdot 10^{-3}$

In the leading log approximation

$$\delta^{(3)}_{\mathrm{LLA}} = (L-1)^3 \left(\frac{\alpha}{\pi}\right)^3 \frac{1}{6} \left(P^{(0)} \otimes P^{(0)} \otimes P^{(0)}\right)_\Delta,$$

$$\left(P^{(0)} \otimes P^{(0)} \otimes P^{(0)}\right)_\Delta = 8 \left(P^{(0)}_\Delta\right)^3 - 24\zeta(2)P^{(0)}_\Delta + 16\zeta(3)$$

$$\Rightarrow \quad \delta^{(3)}_{\mathrm{cut}} = (L-1)^3 \left(\frac{\alpha}{\pi}\right)^3 \left[-4\zeta(2)P^{(0)}_\Delta + \frac{8}{3}\zeta(3)\right]$$

which is not small and reaches $2 \cdot 10^{-3}$

Light pair corrections

A quick estimate can be done within LLA:

$$\delta^{LLA}_{\rm pair} = \frac{2}{3}\left(\frac{\alpha}{2\pi}L\right)^2 P^{(0)}_{\Delta} + \frac{4}{3}\left(\frac{\alpha}{2\pi}L\right)^3 \left\{\left(P^{(0)}\otimes P^{(0)}\right)_{\Delta} + \frac{2}{9}P^{(0)}_{\Delta}\right\} + \mathcal{O}\left(\alpha^2 L, \alpha^4 L^4\right)$$

$$P^{(0)}_{\Delta} = 2\ln\Delta + \frac{3}{2}, \qquad \left(P^{(0)}\otimes P^{(0)}\right)_{\Delta} = \left(P^{(0)}_{\Delta}\right)^2 - \frac{\pi^2}{3}$$

The energy of the emitted pair is limited by the same parameter: $E_{\rm pair} \leq \Delta E$. Both virtual and real e^+e^- pair corrections are taken into account.

Typically, $\mathcal{O}\left(\alpha^2\right)$ pair RC are a few times less than $\mathcal{O}\left(\alpha^2\right)$ photonic ones, see e.g. A.A. JHEP'2001

Complete NLLA corrections (I)

The NLO structure function approach for QED was first applied in F.A. Berends et al. NPB'1987, and then developed in A.A. & K.Melnikov PRD'2002.

The master formula for *ep* scattering reads

$$\mathrm{d}\sigma = \int_{\bar{z}}^{1} \mathrm{d}z \mathcal{D}_{ee}^{\mathrm{str}}(z)\left(\mathrm{d}\sigma^{(0)}(z) + \mathrm{d}\bar{\sigma}^{(1)}(z) + \mathcal{O}\left(\alpha^2 L^0\right)\right) \int_{\bar{y}}^{1} \frac{\mathrm{d}y}{Y} \mathcal{D}_{ee}^{\mathrm{frg}}\left(\frac{y}{Y}\right)$$

where $\mathrm{d}\bar{\sigma}^{(1)}$ is the $\mathcal{O}(\alpha)$ correction to the *ep* scattering with a "massless electron" in the $\overline{\mathrm{MS}}$ scheme

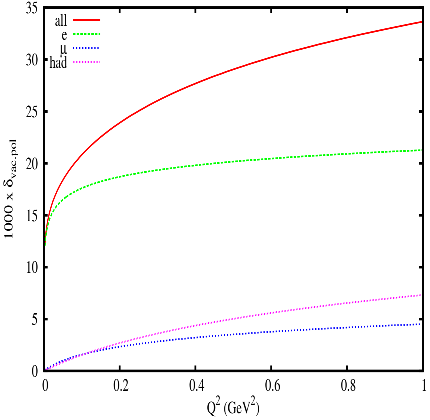
Complete NLLA corrections (II)

$$
\begin{aligned}
\mathrm{d}\sigma^{\mathrm{NLO}} &= \int_{1-\Delta}^{1} \mathcal{D}_{ee}^{\mathrm{str}} \otimes \mathcal{D}_{ee}^{\mathrm{frg}}(z) \left[\mathrm{d}\sigma^{(0)}(z) + \mathrm{d}\bar{\sigma}^{(1)}(z) \right] \mathrm{d}z \\
&= \mathrm{d}\sigma^{(0)}(1) \left\{ 1 + 2\frac{\alpha}{2\pi} \left[L P_{\Delta}^{(0)} + (d_1)_{\Delta} \right] + 2\left(\frac{\alpha}{2\pi}\right)^2 \left[L^2 \left(P^{(0)} \otimes P^{(0)} \right)_{\Delta} \right.\right. \\
&\left.\left. + \frac{1}{3} L^2 P_{\Delta}^{(0)} + 2L(P^{(0)} \otimes d_1)_{\Delta} + L(P_{ee}^{(1,\gamma)})_{\Delta} + L(P_{ee}^{(1,\mathrm{pair})})_{\Delta} \right] \right\} \\
&+ \mathrm{d}\bar{\sigma}^{(1)}(1)\, 2\frac{\alpha}{2\pi} L P_{\Delta}^{(0)} + \mathcal{O}\left(\alpha^3 L^3\right)
\end{aligned}
$$

$$
(d_1)_{\Delta} = -2\ln^2\Delta - 2\ln\Delta + 2, \quad \ldots
$$

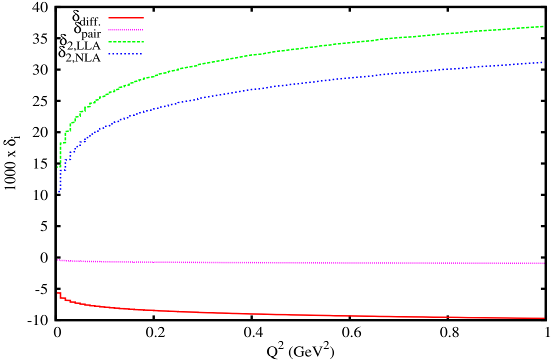
N.B. Th method gives complete $\mathcal{O}\left(\alpha^2 L\right)$ results for sufficiently inclusive observables.

Numerical results: vacuum polarization



Vacuum polarization corrections due to electrons (e), muons (μ), hadrons (had), and the combined effect (all).

Numerical results: RC to electron line



$$\delta_i = \mathrm{d}\sigma^{(i)}/\mathrm{d}\sigma^{(0)}$$

$$\delta_{\mathrm{diff.}} = \frac{\mathrm{d}\sigma^{\mathrm{NLO}}}{\mathrm{d}\sigma^{(0)}} + \delta^{(3)}_{\mathrm{LLA}} + \delta^{(3)}_{\mathrm{LLA,pair}} + \delta^{(4)}_{\mathrm{LLA}} - \exp\{\delta^{(1)}\}$$

New experiment is proposed at MAMI

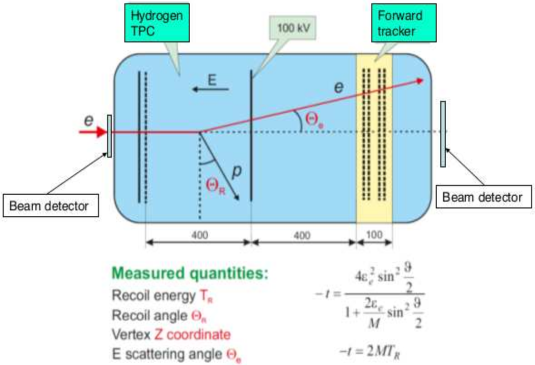
Proposal to perform an experiment at the A2 hall, MAMI:

High Precision Measurement of the ep elastic cross section at small Q^2

Contact persons for the Experiment:

Alexey Vorobyev, Petersburg Nuclear Physics Institute

Achim Denig, Institute for Nuclear Physics, JGU Mainz



Summary for Part II

1. There are several remarks to application of RC in the analysis of MAMI data

2. An advanced treatment of higher order QED RC to the electron line is suggested

3. In particular, effects due to multiple radiation and pair emission in the LLA and NLLA are calculated

4. It is shown that vacuum polarization by muons and hadrons should be taken into account

5. The size of the higher order effects make them relevant for the high-precision experiments

6. Higher order RC to the electron line should be combined with an advanced treatment of two-photon exchange and other relevant effects